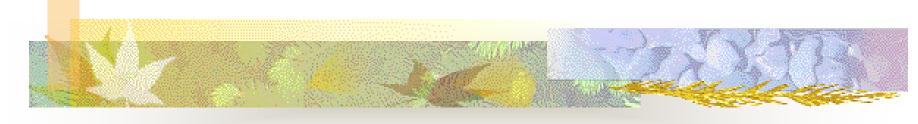
Section-D



Laplace Transform (LT) and Z-Transform

Introduction

- Introduced by Pierre Simon Marquis de Laplace (1749-1827). Practically used by Oliver Heaviside (1850-1925)
- Transform relationship between x(t) and X(s):

Laplace Transform
$$X(t) \longrightarrow X(s)$$
Inverse Laplace Transform

Laplace Transform Pairs

- The Laplace transform originated from the Fourier transform.
- Definition:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt$$

$$x(t) = \frac{1}{2\pi i} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s)e^{st} ds$$

where s is a complex variable that is defined as $s = \sigma + jw$.

Laplace transform

•Definition:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

$$x(t) = \frac{1}{2\pi j} \int_{-\sigma - j\omega}^{\sigma + j\omega} X(s)e^{+st}ds$$

• For a causal system, where $t \ge 0$, the Laplace Transform will be:

$$X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$$

$$\sigma + j\omega$$

$$x(t) = \int_{0}^{\sigma + j\omega} X(s)e^{+st}ds$$

• The Laplace transform originated from the Fourier transform.

$$X(s)_{|s=\sigma+j2\pi f} = \int_{0}^{\infty} x(t)e^{-(\sigma+j2\pi f)t}dt$$

$$X(\sigma + j2\pi f) = \int_{0}^{\infty} \left[x(t)e^{-\sigma t} \right] e^{-j2\pi ft} dt$$

$$\int_{0}^{\infty} y(t) e^{-j2\pi ft} dt$$

Examples of Laplace Transform

•Example 1:

A signal is defined as

$$x(t)=1$$
 $t\ge 0$
= 0 elsewhere

$$X(s) = \int_{0}^{\infty} x(t)e^{-st}dt = \int_{0}^{\infty} (1)e^{-st}dt$$
$$= \frac{1}{s}$$

•Example 2:

A signal is defined as

$$x(t) = e^{-at} \qquad t > 0$$
$$= 0 \qquad t < 0$$

$$X(s) = \int_{0}^{\infty} x(t)e^{-st}dt = \int_{0}^{\infty} \exp(-at)e^{-st}dt$$

$$= \int_{0}^{\infty} e^{-(a+s)t}dt = -\frac{e^{-(a+s)t}}{(a+s)}\Big|_{0}^{\infty}$$

$$= -\frac{e^{-(a+s)\infty}}{(a+s)} + \frac{\exp^{-(a+s)0}}{(a+s)} = \frac{1}{(s+a)}$$

•Example 3

A sinusoidal signal is expressed as

$$x(t) = \cos(2\pi f_1 t) \qquad t \ge 0$$

$$= 0 \qquad t < 0$$

$$X(s) = \int_{0}^{\infty} x(t)e^{-st}dt = \int_{0}^{\infty} \cos(2\pi f_1 t)e^{-st}dt$$

By defining the cosine function in terms of complex exponentials, the Laplace transform is

$$X(s) = \frac{s}{\left(s^2 + \left(2\pi f_1\right)^2\right)}$$